The authors are searching for a dictionary to translate notions from geometry of algebraic curves over finite fields into notions from noncommutative geometry of the adèles class space, aiming to prove the Riemann hypothesis.

Authors’ summary: This paper explores analogies between the Weil proof of the Riemann hypothesis for function fields and the geometry of the adèles class space, which is the noncommutative space underlying Connes’ spectral realization of the zeros of the Riemann zeta function. We consider the cyclic homology of the cokernel (in the abelian category of cyclic modules) of the “restriction map” defined by the inclusion of the idèles class group of a global field in the noncommutative adèles class space. Weil’s explicit formula can then be formulated as a Lefschetz trace formula for the induced action of the idèles class group on this cohomology. In this formulation the Riemann hypothesis becomes equivalent to the positivity of the relevant trace pairing. This result suggests a possible dictionary between the steps in the Weil proof and corresponding notions involving the noncommutative geometry of the adèles class space, with good working notions of correspondences, degree, and codegree etc. In particular, we construct an analog for number fields of the algebraic points of the curve for function fields, realized here as classical points (low temperature KMS states) of quantum statistical mechanical systems naturally associated to the periodic orbits of the action of the idèles class group, that is, to the noncommutative spaces on which the geometric side of the trace formula is supported.

For the entire collection see [Zbl 1185.00041].

Reviewer: Florin Nicolae (Berlin)