Anco, Stephen C.; Asadi, Esmaeel
Quaternionic soliton equations from Hamiltonian curve flows in $\mathbb{H}\mathbb{P}^n$. (English)

In some previous papers of the first author, as [SIGMA, 2, Paper 044, 17 p. (2006; Zbl 1102.37042), J. Geom. Phys. 58, No. 1, 1–37 (2008; Zbl 1134.53027)] and others, the soliton equations encoded using geometric objects, as flows of curves on certain homogeneous spaces $G/H$ are studied and bi-Hamiltonian structures that generate hierarchies of integrable curve flows are considered. In the same line, in the paper under review the author obtains a bi-Hamiltonian geometric curve flows in

$$IHP^n \simeq U(n+1, IH)/U(1, IH) \times U(n, IH) \simeq Sp(n+1)/Sp(1) \times Sp(n).$$

Scalar-vector (multi-component) versions of the sine-Gordon and the modified Korteweg-de Vries equations are obtained and studied via the bi-Hamiltonian structures and conservation laws.

Reviewer: Paul Popescu (Craiova)

MSC:

37K10 Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)
35Q51 Soliton equations
35Q53 KdV equations (Korteweg-de Vries equations)
35Q55 NLS equations (nonlinear Schrödinger equations)
37K05 Hamiltonian structures, symmetries, variational principles, conservation laws (MSC2010)
37K25 Relations of infinite-dimensional Hamiltonian and Lagrangian dynamical systems with topology, geometry and differential geometry
37K40 Soliton theory, asymptotic behavior of solutions of infinite-dimensional Hamiltonian systems
53C35 Differential geometry of symmetric spaces
53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions
35L60 First-order nonlinear hyperbolic equations
35A30 Geometric theory, characteristics, transformations in context of PDEs

Keywords:
quaternion; soliton; Hamiltonian curve flow; symmetric space; hierarchy of Hamiltonian vector fields; SG equation; mKdV equation; evolution equation

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