Grooten, M. I.; Steenbrink, J. H. M.
Quartic double solids with ordinary singularities. (English) Zbl 1209.14012
Proc. Steklov Inst. Math. 267, 104-112 (2009).

A double solid is a double cover of $\mathbb{P}^3$ branched along a surface of even degree. This paper deals with *ordinary* double solids, which are those where the ramification surface has at worst ordinary singularities. In local holomorphic coordinates u, v, w, t, the singularities of such a double solid are thus of three types: $t^2 = uv$ (type A), $t^2 = uvw$ (type T), and $t^2 = u^2 - vw^2$ (type D). The authors describe the mixed Hodge structures on the homology groups of the more general class of ADT threefolds X: they are pure with the exception of $H^3(X)$, which is an extension of a Hodge structure of weight 3 by a Hodge structure of type $(1,1)$, with extension data determined by the Abel-Jacobi mapping to the intermediate Jacobian of the natural resolution of singularities of X. They then study in detail the cyclide double solid (ramified along an irreducible quartic surface whose singular locus is a smooth plane conic), and show that the Torelli mapping, sending X to the polarized mixed Hodge structure on $H_3(X)$, is six-to-one.

Reviewer: Christian Schnell (Chicago)

MSC:

14E20 Coverings in algebraic geometry
14J30 3-folds
14C30 Transcendental methods, Hodge theory (algebro-geometric aspects)

Keywords:

double solid; ordinary singularities; Torelli theorem; ADT threefold

Full Text: DOI arXiv

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