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Euler's elastica and beyond. (English) Zbl 1209.37086

J. Geom. Symmetry Phys. 17, 45-86 (2010).

In 1691, James (Jacob) Bernoulli proposed the elastica problem, which was essentially solved by Euler in 1744: What shape can be obtained when an elastica, an ideal infinitesimally thin elastic rod, is bent without stretching on a plane?

The author reviews the studies and classification of Bernoulli and Euler from an historic point of view and indicates connections with elliptic curves, lemniscate, nonlinear integrable differential equations, etc.

The second part of the paper summarizes the author's work over two decades on the quantized elastica problem, i.e., statistical mechanics of elastica in a heat bath, which serves as a model of the DNA. In this context the modified Korteweg–de Vries hierarchy, loop space, and the submanifold Dirac operator appear.

Reviewer: Johanna Michor (Wien)

MSC:

37K20	Relations of infinite-dimensional Hamiltonian and Lagrangian dynamical systems with algebraic geometry, complex analysis, and special functions
82D60	Statistical mechanics of polymers
37K10	Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)
35Q53	KdV equations (Korteweg-de Vries equations)
74G65	Energy minimization in equilibrium problems in solid mechanics
58E50	Applications of variational problems in infinite-dimensional spaces to the sciences
74K10	Rods (beams, columns, shafts, arches, rings, etc.)

Cited in **7** Documents

Keywords:

Euler's elastica; lemniscate; quantized elastica; mKdV hierarchy

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