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Nonlinear pseudodifferential equations on a segment. (English) Zbl 1212.35417

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Summary: We study the global existence and large-time asymptotic behavior of solutions to the initial/boundary-value problem for the nonlinear nonlocal Whitham equation on a segment $(0, a)$,

$$\begin{cases} u_t + uu_x + \mathbb{K}u = 0, & t > 0,\ x \in (0, a) \\ u(x, 0) = u_0(x), & x \in (0, a), \end{cases}$$

where the pseudodifferential operator $\mathbb{K}u$ on a segment $[0, a]$ is defined by

$$\mathbb{K}u = \theta_a(x) \frac{1}{2\pi \mathrm{i}} \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \mathrm{e}^{px} K(p) \Big(\widehat{u}(p, t) - \frac{u(0, t) - \mathrm{e}^{-pa} u(a, t)}{p} \Big) \, \mathrm{d}p, \tag{1}$$

where $K(p) = C_\alpha p^\alpha$, $\alpha \in (\frac{3}{2}, 2)$, and C_α is chosen by the dissipation conditions. We prove that if the initial data $u_0 = \mathbf{L}^\infty(0, a)$ have a small norm $\|u_0\|_{\mathbf{L}^\infty} < \varepsilon$, then there exists a unique solution $u \in \mathbf{C}([0, \infty); \mathbf{L}^2(0, a)) \cap \mathbf{C}((0, \infty); \mathbf{H}^1(0, a))$ to problem (1). Moreover, there exists a function $A(x) \in \mathbf{L}^\infty(0, a)$ such that the solution has the following asymptotics for large time $t \to \infty$:

$$u(x, t) = A(x) B t^{-\frac{1}{\alpha}} + O(t^{-\frac{1+\delta}{\alpha}}),$$

uniformly with respect to $x \in (0, a)$, where $\delta \in (0, 2 - \alpha)$.

MSC:

35Q53 KdV equations (Korteweg-de Vries equations)
35S15 Boundary value problems for PDEs with pseudodifferential operators
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

Whitham equation