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Homeomorphism with zero Jacobian: sharp integrability of the derivative. (English)


It is shown that for $N \geq 2$ there is a homeomorphism $f$ of $[0,1]^N$ of class $W^{1,p}$ for all $p \in [1,N)$, even

$$\sup_{0<\varepsilon \leq N-1} \varepsilon \int_{[0,1]^N} |Df|^{N-\varepsilon} < \infty,$$

such that $f$ is the identity on the boundary of $[0,1]_N$ but $\det Df(x) = 0$ almost everywhere. Recall that if $f \in W^{1,1}$ is a homeomorphism of $[0,1]^N$ with $\det Df(x) = 0$ a.e. then $f$ maps a null set onto a set of full measure by the area formula of P. Hajłasz [Colloq. Math. 64, 93–101 (1993; Zbl 0840.26009)]. It is known that no homeomorphism $f$ of $[0,1]^N$ can satisfy

$$\lim_{\varepsilon \to 0^+} \varepsilon \int_{\Omega} |Df|^{N-\varepsilon} = 0,$$

since $f$ would then map null sets to null sets [see J. Kauhanen, P. Koskela and J. Malý, Mich. Math. J. 49, 169–181 (2001; Zbl 0997.30018)]. Hence, the obtained integrability condition for $|Df|$ is almost the best one can expect for such a map $f$.

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MSC:
26B10 Implicit function theorems, Jacobians, transformations with several variables
26B20 Integral formulas of real functions of several variables (Stokes, Gauss, Green, etc.)
26B30 Absolutely continuous real functions of several variables, functions of bounded variation

Keywords:
homeomorphism; Luzin condition; Jacobian

Full Text: DOI

References:
[2] Hencl, S., Sobolev homeomorphism with zero Jacobian almost everywhere, submitted for publication, preprint is available at Zbl 1222.26018

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