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Singular Levi-flat hypersurfaces and codimension one foliations. (English) Zbl 1214.32012

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A Levi-flat hypersurface M in a complex manifold X is a smooth real hypersurface such that the field of its tangent complex hyperplanes forms an integrable distribution, i.e., a real hypersurface foliated by complex hypersurfaces. The latter foliation is called the Levi foliation. A theorem of E. Cartan says that, if M is real analytic and smooth, then the Levi foliation always extends to a holomorphic foliation by hypersurfaces on a complex neighborhood of M.

The paper under review extends Cartan's theorem to singular real analytic Levi-flat hypersurfaces $M \subset X$. Namely, Theorem 1.3 of the paper says that there always exist another complex manifold Y of the same dimension as X and holomorphically projected onto X, and a real analytic Levi-flat hypersurface $N \subset Y$ that is extendable to a (singular) holomorphic foliation by hypersurfaces on Y such that (1) the projection sends an open subset $N_0 \subset N$ isomorphically onto the regular part of the initial singular Levi-flat hypersurface M, and (2) the restriction to $\overline{N}_0$ of the projection is a proper map onto the closure of the regular part of M.

This is the first step towards desingularization of Levi flat hypersurfaces. It is a remarkable result that will have important applications in holomorphic foliations, complex analysis and geometry.

The proof is based on the following nice idea: to lift the Levi foliation to the projectivized cotangent bundle and prove the result for the lifted Levi-flat surface (which now has codimension greater than 1 in the ambient complex manifold). The key argument is given by Theorem 2.5, which deals with a (singular) Levi-flat real analytic subset N of arbitrary codimension in a complex manifold. Let N_{reg} denote the regular part of N. Theorem 2.5 says that every point of the closure $\overline{N_{\text{reg}}}$ has a neighborhood where $\overline{N_{\text{reg}}}$ is locally a Levi-flat hypersurface in some complex submanifold (whose real dimension is thus equal to $\dim N + 1$).

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MSC:

32V25	Extension of functions and other analytic objects from CR manifolds
32S65	Singularities of holomorphic vector fields and foliations
32C05	Real-analytic manifolds, real-analytic spaces

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complex manifold; (singular) Levi-flat hypersurface; holomorphic foliation; resolution of singularities

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