**Hančl, Jaroslav**; **Schinzel, Andrzej**; **Šustek, Jan**
**On expressible sets of geometric sequences.** (English) Zbl 1215.11077
Funct. Approximatio, Comment. Math. 39, Part 1, 71-95 (2008).

The paper under review is concerned with the set of real numbers $x$ which can be expressed in the form

$$x = \sum_{n=1}^{\infty} \frac{1}{c_n A^n},$$

where $A > 3$ is a real number and $\{c_n\}$ is some sequence of natural numbers. The set of such numbers is called the expressible set of the geometric sequence $A^n$.

It is shown that this set is Borel and contains the interval $(0, 1/((A-1)(\lceil A \rceil - 2))]$, and upper and lower bounds on the Lebesgue measure of the set are obtained. In the case when $A = 4$, the upper and lower bounds coincide, and the measure of the expressible set is equal to $1/4$. In this case, the interval shown to be contained in the set is equal to $(0, 1/6]$. Finally, the order at which the Lebesgue measure of the expressible set tends to zero as $A$ increases is studied. It is shown that a lower bound decays like $A^{-2}$, while an upper bound decays like $A^{-3/2}$. The exact asymptotic decay remains an open problem.

Reviewer: Simon Kristensen (Aarhus)

**MSC:**

11K55 Metric theory of other algorithms and expansions; measure and Hausdorff dimension

Cited in **1** Document

**Keywords:**

Expressible sets; geometric sequences; Lebesgue measure.

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