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A fixed-point theorem for integral type inequality depending on another function. (English)

Zbl 1216.54015
Int. J. Math. Anal., Ruse 4, No. 29-32, 1491-1499 (2010).

Let (X,d) be a complete metric space, $0 \leq \alpha < 1$, and S,T be selfmaps of X such that T is injective, continuous, and subsequentially convergent. Suppose that there exists $x \in X$ such that $\int_0^{d(Tsy,TS^2y)} \varphi(t)\,dt \leq \alpha \int_0^{d(T,TSy)} \varphi(t)\,dt$ for each y in the orbit of x, where $\varphi := [0,+\infty) \to [0,+\infty)$ is a Lebesgue integrable mapping which is summable, nonnegative, and such that $\int_0^{\varepsilon} \varphi(t)\,dt > 0$ for each $\varepsilon > 0$. Then the authors show that

(i) $\lim_n TS^n x = Tq$,

(ii) $\int_0^{d(Tq,TS^n x)} \varphi(t)\,dt \leq \alpha_n \int_0^{d(Tq,Tx)} \varphi(t)\,dt$, and

(iii) q is a fixed point of S if and only if $G(x) := d(TSx,Tx)$ is S_T-orbitally lower semicontinuous at q.

Reviewer: Billy E. Rhoades (Bloomington)

MSC:

54H25 Fixed-point and coincidence theorems (topological aspects)

Keywords:

contraction mapping; contractive mapping; fixed point; ingegral type; subsequentially convergent

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