Structured derivations were introduced by R.-J. Back and J. von Wright [Refinement calculus. A systematic introduction, New York: Springer (1998; Zbl 0949.68094)] as an extension of the calculational proof style originally proposed by E. W. Dijkstra and his colleagues. Structured derivations added nested subderivations and inherited assumptions to this style. This paper introduces further extensions of the structured derivation format, and gives a precise syntax and semantics for the extended proof style. The extensions provide a unification of the three main proof styles used in mathematics today: Hilbert-style forward chaining proofs, Gentzen-style backward chaining proofs and algebraic derivations and calculations (in particular, Dijkstra’s calculational proof style). Each of these proof styles can now be directly presented as a structured derivation. Even more importantly, the three proof styles can be freely intermixed in a single structured derivation, allowing different proof styles to be used in different parts of the derivation, each time choosing the proof style that is most suitable for the (sub)problem at hand. We describe here (extended) structured derivations, feature by feature, and illustrate each feature with examples. We show how to model the three main proof styles as structured derivations. We give an exact syntax for structured derivations and define their semantics by showing how a structured derivation can be automatically translated into an equivalent Gentzen-style sequent calculus derivation. Structured derivations have been primarily developed for teaching mathematics at the secondary and tertiary education level. The syntax of structured derivations determines the general structure of the proof, but does not impose any restrictions on how the basic notions of the underlying mathematical domain are treated. Hence, the style can be used for any kind of proofs, calculations, derivations, and general problem solving found in mathematics education at these levels. The precise syntax makes it easy to provide computer support for structured derivations.

MSC:
03F03 Proof theory in general (including proof-theoretic semantics)
00A35 Methodology of mathematics
03B35 Mechanization of proofs and logical operations
97E40 Language of mathematics (educational aspects)
97E50 Reasoning and proving in the mathematics classroom

Keywords:
structured derivations; proof styles; Gentzen-style proofs; Hilbert-style proofs; Dijkstra’s calculational proofs; teaching mathematics; high-school mathematics; proof format; sequent calculus

Software:
Yices; SIMPLIFY; HOL; Isar; PVS

Full Text: DOI

References:


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