Dimakis, Aristophanes; Müller-Hoissen, Folkert
Bidifferential calculus approach to AKNS hierarchies and their solutions. (English)
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Summary: We express AKNS hierarchies, admitting reductions to matrix NLS and matrix mKdV hier-
archies, in terms of a bidifferential graded algebra. Application of a universal result in this framework
quickly generates an infinite family of exact solutions, including e.g. the matrix solitons in the focusing
NLS case. Exploiting a general Miura transformation, we recover the generalized Heisenberg magnet hi-
erarchy and establish a corresponding solution formula for it. Simply by exchanging the roles of the two
derivations of the bidifferential graded algebra, we recover “negative flows”, leading to an extension of the
respective hierarchy. In this way we also meet a matrix and vector version of the short pulse equation
and also the sine-Gordon equation. For these equations corresponding solution formulas are also derived.
In all these cases the solutions are parametrized in terms of matrix data that have to satisfy a certain
Sylvester equation.

MSC:
37J35 Completely integrable finite-dimensional Hamiltonian systems, inte-
   gration methods, integrability tests
37K10 Completely integrable infinite-dimensional Hamiltonian and La-
   grangian systems, integration methods, integrability tests, integrable
   hierarchies (KdV, KP, Toda, etc.)
16E45 Differential graded algebras and applications (associative algebraic as-

Keywords:
AKNS hierarchy; negative flows; Miura transformation; bidifferential graded algebra; Heisenberg magnet;
mKdV; NLS; sine-Gordon; vector short pulse equation; matrix solitons

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