Summary: Fix strictly increasing right continuous functions with left limits and periodic increments, \( W_i : \mathbb{R} \to \mathbb{R}, i = 1, \ldots, d \), and let \( W(x) = \sum_{i=1}^{d} W_i(x_i) \) for \( x \in \mathbb{R}^d \). We construct the \( W \)-Sobolev spaces, which consist of functions \( f \) having weak generalized gradients \( \nabla_W f = (\partial_{W_1} f, \ldots, \partial_{W_d} f) \). Several properties, that are analogous to classical results on Sobolev spaces, are obtained. Existence and uniqueness results for \( W \)-generalized elliptic equations, and uniqueness results for \( W \)-generalized parabolic equations are also established. Finally, an application of this theory to stochastic homogenization is presented.

MSC:

35A23  Inequalities applied to PDEs involving derivatives, differential and integral operators, or integrals
35J15  Second-order elliptic equations
35K10  Second-order parabolic equations
46E35  Sobolev spaces and other spaces of “smooth” functions, embedding theorems, trace theorems

Keywords:
Poincaré inequality; compact embedding; stochastic homogenization

Full Text: DOI

References:


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