Summary: The study of Borel equivalence relations under Borel reducibility has developed into an important area of descriptive set theory. The dichotomies of J. H. Silver [Ann. Math. Logic 18, 1–28 (1980; Zbl 0517.03018)] and L.-A. Harrington, A. S. Kechris and A. Louveau [J. Am. Math. Soc. 3, No. 4, 903–928 (1990; Zbl 0778.28011)] show that, with respect to Borel reducibility, any Borel equivalence relation strictly above equality on $\omega$ is above equality on $\mathcal{P}(\omega)$, the power set of $\omega$, and any Borel equivalence relation strictly above equality on the reals is above equality modulo finite on $\mathcal{P}(\omega)$. In this article we examine the effective content of these and related results by studying effectively Borel equivalence relations under effectively Borel reducibility. The resulting structure is complex, even for equivalence relations with finitely many equivalence classes. However, use of Kleene’s $O$ as a parameter is sufficient to restore the picture from the noneffective setting. A key lemma is that of the existence of two effectively Borel sets of reals neither of which contains the range of the other under any effectively Borel function; the proof of this result applies Barwise compactness to a deep theorem of Harrington establishing for any recursive ordinal $\alpha$ the existence of $\Pi^0_1$ singletons whose $\alpha$-jumps are Turing-incomparable.

MSC:

- 03E15 Descriptive set theory
- 03D30 Other degrees and reducibilities in computability and recursion theory
- 03D60 Computability and recursion theory on ordinals, admissible sets, etc.

Keywords:

hyperarithmetic equivalence relations; Borel reducibility; dichotomy theorems

Full Text: DOI

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