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Schrödinger equation with critical Sobolev exponent. (English) Zbl 1224.35126


Let $N \geq 3$, $1 < p < \sigma = \frac{N+2}{N-2}$. Let the problem of solvability of

\[-\varepsilon^2 \Delta u + V(x)u = K(x)u^p + Q(x)u^\sigma \quad \text{on } \mathbb{R}^N, \quad u > 0 \quad \text{on } \mathbb{R}^N, \quad \lim_{|x| \to \infty} u(x) = 0\]

be formulated with the assumptions: $V, K, Q \in C^2(\mathbb{R}^N)$, $V, K, D^2 V, D^2 K, D^2 Q$ bounded on $\mathbb{R}^N$, $V(x) \geq C > 0$, $K(x) \geq C' > 0$ on $\mathbb{R}^N$ and $Q(0) = 0$.

In the paper, the existence theorem is proved. In the case $K \equiv V \equiv 1$ it sounds (Corollary 1.2):

Let $\xi_0 \in \mathbb{R}^N$. There exists $\varepsilon_0$ such that if $\varepsilon \in (0, \varepsilon_0)$ then the problem has a solution $u_\varepsilon$ which concentrates on $\xi_\varepsilon$ with $\lim_{\varepsilon \to 0^+} \xi_\varepsilon = \xi_0$, provided that one of the two following conditions holds:

(a) $\xi_0$ is a nondegenerate critical point of $Q$,

(b) $\xi_0$ is a point of an isolated strict local minimum or maximum of $Q$.

The new feature of the present paper is that the coefficient $Q$ of $u^\sigma$ vanishes at $x = 0$. After the rescaling $x \mapsto \varepsilon x$ the equation becomes $-\Delta u + V(\varepsilon x)u = K(\varepsilon x)u^p + Q(\varepsilon x)u^\sigma$. Then the assumption $Q(0) = 0$ implies that the unperturbed problem with $\varepsilon = 0$ is unaffected by the critical nonlinearity.

The proof of the main result is based on the fact that the term with critical growth $(Q(x)u^\sigma)$ is a small perturbation of the problem without it. Using that, the author can consider as a main part of the problem its finite dimensional part obtained by Lyapunov-Schmidt reduction.

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MSC:

35J60 Nonlinear elliptic equations
35B25 Singular perturbations in context of PDEs
35B33 Critical exponents in context of PDEs
35J20 Variational methods for second-order elliptic equations
58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel’man theory, etc.) in infinite-dimensional spaces

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semilinear Schrödinger equation

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