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Schanuel's conjecture and algebraic powers z^w and w^z with z and w transcendental. (English)

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The authors prove several conditional results on transcendence assuming Schanuel's conjecture. The conjecture states that if numbers $\alpha_1, \dots, \alpha_n \in \mathbb{C}$ are linearly independent over \mathbb{Q} then there are at least n algebraically independent numbers among $\alpha_1, \dots, \alpha_n, e^{\alpha_1}, \dots, e^{\alpha_n}$.

Under this conjecture the authors prove that if for $z, w \in \mathbb{C} \setminus \{0, 1\}$ the numbers z^w and w^z are algebraic, then z and w are either both rational or both transcendental. This, for instance, implies the transcendence of the numbers i^{e^π} and i^{i^i} .

Another conditional result is that if $\alpha \neq 0$ and z are complex numbers with α algebraic and z irrational such that $\alpha^{\alpha^z} = z$, then z is transcendental.

Reviewer: Jan Šustek (Ostrava)

MSC:

11J81 Transcendence (general theory)

Cited in 1 Document

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Schanuel's conjecture; transcendence; algebraic independence

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