Sondow, Jonathan; Schalm, Kyle

Which partial sums of the Taylor series for e are convergents to e? (and a link to the primes 2, 5, 13, 37, 463). II. (English) Zbl 1227.11031

Amdeberhan, Tewodros (ed.) et al., Gems in experimental mathematics. AMS special session on experimental mathematics, Washington, DC, January 5, 2009. Providence, RI: American Mathematical Society (AMS) (ISBN 978-0-8218-4869-2/pbk). Contemporary Mathematics 517, 349-363 (2010).

Let the nth partial sum of the Taylor series $e = \sum_{r=0}^{\infty} \frac{1}{r!}$ be $\frac{A_n}{n!}$, and let $\frac{p_k}{q_k}$ be the kth convergent of the simple continued fraction for e. The authors prove that given any three consecutive partial sums s_n, s_{n+1}, s_{n+2} at most two of them are convergents to e. For any positive integer k, there exists a constant $n(k)$ such that if $n \geq n(k)$, then among the k consecutive partial sums $s_n, s_{n+1}, \dots, s_{n+k-1}$ at most two are convergents to e. Almost all partial sums are not convergents to e. A related result about the denominators q_k and powers of factorials is proved. There is a connection between the A_n and the primes $2, 5, 13, 37, 463$.

For Part I see Contemp. Math. 457, 273–284 (2008; Zbl 1159.11004).

For the entire collection see [Zbl 1193.00060].

Reviewer: Florin Nicolae (Berlin)

MSC:

11A55 Continued fractions
11A41 Primes
11Y55 Calculation of integer sequences
11Y60 Evaluation of number-theoretic constants

Keywords:

Taylor series; partial sum; simple continued fraction for e; convergent

Software:

SageMath

Full Text: arXiv