Chung, Ping-Tsai; Lee, Sin-Min
A note on the edge-balance index sets for graphs. (English) Zbl 1229.05241

Summary: Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $Z_2 = \{0, 1\}$. An edge labeling $f : E(G) \to Z_2$ of a graph $G$ is said to be edge-friendly if $|e_f(0) - e_f(1)| \leq 1$, where $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. An edge-friendly labeling $f : E(G) \to Z_2$ induces a partial vertex labeling $f^+ : V(G) \to A$ defined by $f^+(x) = 0$ if the number of edges incident with $x$ having edge label of 0 is greater than the number of edges incident with $x$ having edge label of 1. Similarly, $f^+(x) = 1$ if the number of edges incident with $x$ having edge label of 1 is greater than the number of edges incident with $x$ having edge label of 0. $f^+(x)$ is not defined if the number of edges incident with $x$ having edge label of 1 equals to the number of edges incident with $x$ having edge label of 0. For $i \in Z_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph $G$, $EBI(G)$, is defined as $\{|v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ is edge-friendly}\}$.

Given a base graph $G(V, E)$, the envelope graph $EV(G)$ is the graph with $V(EV(G)) = V(G) \cup E(G)$ and $E(EV(G)) = E(G) \cup \{(u, (u, v)) : u \in V, (u, v) \in E(G)\}$. In this paper, we first discuss the edge-balance index sets of some base graphs $G$ including stars, paths and cycles. We then construct envelope graphs $EV(G)$ from these basic graphs $G$. The motivation to construct envelope graphs $EV(G)$ is to extend some graph structure from the base graphs and to study if the set $EBI(EV(G))$ is related to $EBI(G)$. However, for all base graphs $G$: stars, paths and cycles, we show that $EBI(EV(G))$ is not related to $EBI(G)$. We then formulate a two person edge game: each player in turn labels the edge in turn in a friendly fair way (i.e., edge-friendly), one uses label 0 and the other uses label 1. We count the number of edges incident with each vertex of $G$ having edge label 0 and label 1, respectively. We then label each vertex by $a$ if $a$ is the majority label for all incident edges, where $a \in \{0, 1\}$; otherwise, the vertex is unlabelled. Note that the winner of this two person game is the player holds more vertices having its majority label $a$, where $a \in \{0, 1\}$. Under the edge game consideration, the results of $EBI(G)$ could be interpreted as all possible edge-balance indexes of $G$, where each index denoted as a possible difference of the number of players’ vertex labels.

Finally, we introduce the concept of balance graph which is a dual concept of edge-balance graph. We then show that the computing balance index sets of $G$ is not related to the computing edge-balance index sets of $G$, in general.

MSC:

05C78 Graph labelling (graceful graphs, bandwidth, etc.)
05C38 Paths and cycles
91A05 2-person games

Keywords:

friendly labeling; edge-balance index set; two person game; envelope graphs; balance index set