Summary: A sequence $S = s_1s_2\ldots s_n$ is said to be nonrepetitive if no two adjacent blocks of $S$ are identical. In 1906, A. Thue [Christiania Vidensk.-Selsk. Skr. 1906, Nr. 7, 22 S. (1906; JFM 39.0283.01)] proved that there exist arbitrarily long nonrepetitive sequences over a 3-element set of symbols. We study a generalization of nonrepetitive sequences involving arithmetic progressions. We prove that for every $k \geq 1$, there exist arbitrarily long sequences over at most $2k + 10\sqrt{k}$ symbols whose subsequences, indexed by arithmetic progressions with common differences from the set $\{1, 2, \ldots, k\}$, are nonrepetitive. This improves a previous bound of $e^{33k}$ obtained by the first author. Our approach is based on a technique introduced recently by the first two authors and P. Micek ["A new approach to nonrepetitive sequences" (submitted), arXiv:1103.3809] which was originally inspired by a constructive proof of the Lovász local lemma due to R. A. Moser and G. Tardos ["A constructive proof of the general Lovász local lemma", J. ACM 57, No. 2, Art. No. 11, 15 p. (2010), arXiv:0903.0544]. We also discuss some related problems that can be attacked by this method.

MSC:

- 68R15 Combinatorics on words
- 05D40 Probabilistic methods in extremal combinatorics, including polynomial methods (combinatorial Nullstellensatz, etc.)
- 11B25 Arithmetic progressions

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