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Proofs of power sum and binomial coefficient congruences via Pascal's identity. (English)

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Summary: A well-known and frequently cited congruence for power sums is

$$1^n + 2^n + \cdots + p^n \equiv \begin{cases} -1 \pmod{p} & \text{if } (p-1) \mid n, \\ 0 \pmod{p} & \text{if } (p-1) \nmid n, \end{cases}$$

where $n \geq 1$ and p is prime. We survey the main ingredients in several known proofs. Then we give an elementary proof, using an identity for power sums proven by *B. Pascal* in the year 1654. An application is a simple proof of a congruence for certain sums of binomial coefficients, due to *Ch. Hermite* [J. Reine Angew. Math. 81, 93–95 (1875; JFM 07.0131.01)] and *P. Bachmann* [Niedere Zahlentheorie. Zweiter Teil, Teubner, Leipzig (1910; JFM 41.0221.10) (p. 53); Reprint. Bronx, N. Y.: Chelsea (1968; Zbl 0253.10001)].

MSC:

11A07 Congruences; primitive roots; residue systems
11B65 Binomial coefficients; factorials; q-identities
05A10 Factorials, binomial coefficients, combinatorial functions
05A19 Combinatorial identities, bijective combinatorics

Keywords:

congruence for power sums; congruence for sums of binomial coefficients

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