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Non-Hermitian quantum mechanics. (English) Zbl 1230.81004

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The author shows the theory of the non-Hermitian quantum mechanics (NHQM) in a form with exercises and answers to them. In Exercise 1.4, he treats the Hamiltonian $H = H_0 + \lambda V$ satisfying $H_0(x) = H_0(-x)$ and $V(x) = -V(-x)$. When $\lambda = i\Gamma$ and Γ is real-valued only, H commutes with the PT symmetry operator: $PxP^{-1} = -x$ and $TiT^{-1} = -i$. The eigenvalues of $H(x, \Gamma)$ are real for $|\Gamma| < |\lambda_{bp}|$. As an answer, let $(H_0 + \lambda V)\psi_j = E_j\psi$. A perturbative expansion of ψ_j and E_j in a power series of λ converge for $|\lambda| < |\lambda_{bp}|$. For $\lambda = i\Gamma$ and $\lambda_{bp} = i\Gamma_{bp}$, we can write

$$E(\lambda) = E^{bp} \pm D\{(\lambda - \lambda_{bp})(\lambda - \lambda^*_{bp})\}^{1/2}$$

in the vicinity of λ_{bp}, as the most common situation:

$$E_j(|\lambda| < |\lambda_{bp}|) = \sum_{n=0\sim\infty} \lambda^n E_j^{(n)};\ E_j = \langle\chi_j^{(n)}|H_0 + \lambda V|\chi_j^{(n)}\rangle + 0(\lambda^{2n+2}),$$

$$\chi_j^{(n)}(x) = \sum_{k=0\sim n} \lambda^k \psi_j^{(k)},\ E_j^{(2n)} = \langle\psi_j^{(n)}|V|\psi_j^{(n-1)}\rangle.$$

Since $E_j^{(2n+1)} = 0$, $n = 0, 1, \ldots$, are derived,

$$E_j(|\Gamma| < |\Gamma_{bp}|) = \sum_{n=0\sim\infty} (-1)^n \Gamma^{2n} E_j^{(2n)},$$

has real values only.

The author also shows in Fig.1.1 the metastable resonance states for a particle in a spherically symmetric potential barrier given by $V(r) = (r^2/2 - 0.8)\exp(-0, 1r^2)$. The non-Hermitian methods for the calculation of resonance energies and wave functions are described in Chapters 4–5. As he explains in Chapter 6, the expectation value of a complex scaled dynamical operator $\widehat{0}$ is given by

$$\widetilde{0} = \langle\psi^*|\widehat{0}|\psi\rangle/\langle\psi^*|\psi\rangle \equiv |\widetilde{0}|e^{i\alpha}.$$

The physical interpretations of $\widetilde{0}$ are also given. He treats the examples, like to the one in the above Exercise 1.4, in Chapter 9. That is, at the branch point $\lambda = \lambda_{bp}$, $\psi_1(r; \lambda_{bp}) = \psi_2(r; \lambda_{bp}) \equiv \psi_{bp}$ and $(\psi_{bp}|\psi_{bp}) = 0$ happen. The eigenfunction $\psi_{bp}(r)$ is referred as a self-orthogonal state, and studied from the standpoint of the NHQM. In the last Chapter 10, the points where QM branches into two formalisms (HQM and NHQM) are discussed.

Reviewer: Hideo Yamagata (Osaka)

MSC:

81-02 Research exposition (monographs, survey articles) pertaining to quantum theory
81Q05 Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics
81Q12 Nonselfadjoint operator theory in quantum theory including creation and destruction operators

Cited in **112** Documents

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