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Diffraction of a skew incident plane electromagnetic wave by an impedance wedge. (English)

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Summary: This paper studies one of the canonical problems in the diffraction theory, i.e. determining exactly the field distribution, excited by a skew incident plane electromagnetic wave of arbitrary polarization, outside of a wedge on whose faces boundary conditions of Leontovich type are imposed on the electromagnetic field. To this end, the Sommerfeld-Malyuzhinets technique is used at first to deduce from the impedance type conditions on the wedge faces a matrix difference equation of dimension four for the two spectra (spectral functions) of the field components that are parallel to the edge of the wedge. Eliminating one of the spectra in the matrix functional difference (FD) equation yields a though more complicated but scalar difference equation for the other spectrum. Use is then made of a special function $\chi_\Phi(\alpha)$, a generalization of the eponymous Malyuzhinets function, to transform this difference equation into one with constant coefficients at its left-hand side. Taking into account the asymptotic behavior of the spectrum, originating from the Meixner edge condition, the poles and their respective residues of the spectrum in the basic strip of the complex plane, an integral equivalent to this difference relation turns out in terms of the so-called S-integral. For points on the imaginary axis which belongs to the basic strip in the complex plane, the integral equivalent becomes a Fredholm integral equation of the second kind for the spectrum there. Solving the integral equation by means of, e.g. the quadrature method, the spectrum on the imaginary axis can be obtained, therefore, with help of the integral expression, the spectrum in the basic strip, and outside of it by analytical continuation. A first-order uniform asymptotic solution for points far away from the edge of the wedge follows from an application of the saddle point method. Comparison with available results in several special cases show that this approach leads to a fast and accurate solution of the problem under study. The typical behavior of such a diffraction problem, namely, the depolarization of the diffracted waves for skew incidence, is also demonstrated with examples.

MSC:

78A45 Diffraction, scattering

Keywords:

impedance wedge; electromagnetic wave diffraction; skew incidence; exact and uniform asymptotic solution; Sommerfeld; malyuzhinets technique; matrix functional difference equation; generalized malyuzhinets function χ_ϕ; S-integrals; Fredholm integral equation of the second kind

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