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Global well-posedness of classical solutions with large oscillations and vacuum to the three-dimensional isentropic compressible Navier-Stokes equations. (English) Zbl 1234.35181

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Summary: We establish the global existence and uniqueness of classical solutions to the Cauchy problem for the isentropic compressible Navier-Stokes equations in three spatial dimensions with smooth initial data that are of small energy but possibly large oscillations with constant state as far field, which could be either vacuum or nonvacuum. The initial density is allowed to vanish, and the spatial measure of the set of vacuum can be arbitrarily large; in particular, the initial density can even have compact support. These results generalize previous results on classical solutions for initial densities being strictly away from vacuum and are the first for global classical solutions that may have large oscillations and can contain vacuum states.

MSC:

35Q30 Navier-Stokes equations
76D05 Navier-Stokes equations for incompressible viscous fluids
35A01 Existence problems for PDEs: global existence, local existence, non-existence

Full Text: DOI arXiv Link

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