Santos, Andrés

Solutions of the moment hierarchy in the kinetic theory of Maxwell models. (English)

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Summary: In the Maxwell interaction model the collision rate is independent of the relative velocity of the colliding pair and, as a consequence, the collisional moments are bilinear combinations of velocity moments of the same or lower order. In general, however, the drift term of the Boltzmann equation couples moments of a given order to moments of a higher order, thus preventing the solvability of the moment hierarchy, unless approximate closures are introduced. On the other hand, there exist a number of states where the moment hierarchy can be recursively solved, the solution generally exposing non-Newtonian properties. The aim of this paper is to present an overview of results pertaining to some of those states, namely the planar Fourier flow (without and with a constant gravity field), the planar Couette flow, the force-driven Poiseuille flow, and the uniform shear flow.

MSC:

76P05 Rarefied gas flows, Boltzmann equation in fluid mechanics
82C40 Kinetic theory of gases in time-dependent statistical mechanics

Keywords:

Boltzmann equation; Maxwell molecules; moment hierarchy; non-Newtonian properties

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