Caveney, Geoffrey; **Nicolas, Jean-Louis**; **Sondow, Jonathan**

Robin's theorem, primes, and a new elementary reformulation of the Riemann hypothesis. (English) Zbl 1235.11082

Integers 11, No. 6, 753-763, A33 (2011).

There are equivalent formulations of the Riemann hypothesis (RH) by *G. Robin* (1984, using Euler-constant γ) and *J. C. Lagarias* [Am. Math. Mon. 109, No. 6, 534–543 (2002; Zbl 1098.11005), using harmonic numbers). The authors give another elementary one, using Gronwall's function $G(n) = \frac{\sigma(n)}{n \log \log n}$ $(n > 1)$: RH is true if and only if $n = 4$ is the only composite number with the two properties:

(i) $G(n) \geq G(\frac{n}{p})$ for every prime factor p of n;

(ii) $G(n) \geq G(an)$ for every positive integer a.

The proof is elementary and uses the results of Gronwall and Robin.

Reviewer: Jürgen Spilker (Freiburg i. Br.)

MSC:

11M26	Nonreal zeros of $\zeta(s)$ and $L(s, \chi)$; Riemann and other hypotheses	Cited in **1** Review
11N64	Other results on the distribution of values or the characterization of arithmetic functions	Cited in **2** Documents
11Y55	Calculation of integer sequences	

Keywords:

Riemann hypothesis; Gronwall function; Robin's theorem

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