Matet, Pierre

The Magidor function and diamond. (English) Zbl 1237.03031

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Let κ be a regular uncountable cardinal and λ be a cardinal $> \kappa$. The author shows that (Theorem 4) if $2^{<\kappa} \leq M(\kappa, \lambda)$, then $\diamondsuit_{\kappa,\lambda}$ holds, where $M(\kappa, \lambda)$ is the Magidor function, which is equal to λ^ω if $\mathrm{cf}(\lambda) \geq \kappa$, and $(\lambda^+)^\omega$ otherwise. Here the diamond principle $\diamondsuit_{\kappa,\lambda}$ is a generalization of Jensen's classical diamond, and it asserts that there exists a sequence $\langle s_a : a \in P_\kappa(\lambda)\rangle$ with $s_a \subseteq a$ such that for any $X \subseteq \lambda$, $\{a : s_a = X \cap a\}$ is a stationary subset of $P_\kappa(\lambda)$.

In some sense, this result is optimal due to the well-known fact that, assuming $\diamondsuit_{\kappa,\lambda}$, the least cardinality of any closed unbounded subset of $P_\kappa(\lambda)$, $c(\kappa, \lambda)$, equals $\lambda^{<\kappa}$, and a result of Magidor that, given the nonexistence of ω_1-Erdős cardinals in the core model K, $c(\kappa, \lambda) = M(\kappa, \lambda)$. Therefore, assuming $\diamondsuit_{\kappa,\lambda}$ and there is no ω_1-Erdős cardinal in the core model K, $2^{<\kappa} \leq M(\kappa, \lambda)$.

However, the author also exhibits (relative to a large cardinal) a (Cohen) forcing model in which $2^{<\kappa} > M(\kappa, \lambda)$ and $\diamondsuit_{\kappa,\lambda}$ holds. In fact, a strictly stronger variation of $\diamondsuit_{\kappa,\lambda}$, $\diamondsuit_{\kappa,\lambda}[NG_{\kappa,\lambda}]$, is instead discussed throughout the paper. A further variation of the form $\diamondsuit_{\kappa,\lambda,\lambda}[J]$ is discussed at the end of the paper.

Reviewer: Xianghui Shi (Beijing)

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.