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Ehrhart series of lecture hall polytopes and Eulerian polynomials for inversion sequences.
(English) | Zbl 1237.05017

Summary: For a sequence $s = (s_1, \ldots, s_n)$ of positive integers, an $s$-lecture hall partition is an integer sequence $\lambda$ satisfying $0 \leq \lambda_1/s_1 \leq \lambda_2/s_2 \leq \cdots \leq \lambda_n/s_n$. In this work, we introduce $s$-lecture hall polytopes, $s$-inversion sequences, and relevant statistics on both families. We show that for any sequence $s$ of positive integers:

(i) the $h^*$-vector of the $s$-lecture hall polytope is the ascent polynomial for the associated $s$-inversion sequences;
(ii) the ascent polynomials for $s$-inversion sequences generalize the Eulerian polynomials, including a $q$-analog that tracks a generalization of major index on $s$-inversion sequences; and
(iii) the generating function for the $s$-lecture hall partitions can be interpreted in terms of a new $q$-analog of the $s$-Eulerian polynomials, which tracks a “lecture hall” statistic on $s$-inversion sequences.

We show how four different statistics are related through the three $s$-families of partitions, polytopes, and inversion sequences. Our approach uses Ehrhart theory to relate the partition theory of lecture hall partitions to their geometry.

MSC:
05A17 Combinatorial aspects of partitions of integers

Keywords: lecture hall partitions; Eulerian polynomials; permutation statistics; Ehrhart-theory; inversion sequences; $q$-series identities

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