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[Gelfand, S. I.]

Vector bundles on complex projective spaces. With an appendix by S. I. Gelfand. Corrected reprint of the 1988 edition. (English) Zbl 1237.14003

Modern Birkhäuser Classics. Basel: Birkhäuser (ISBN 978-3-0348-0150-8/pbk; 978-3-0348-0151-5/ebook). viii, 239 p. (2011).

The review of the first edition can be found here [Progress in Mathematics. 3. Boston - Basel - Stuttgart: Birkhäuser. (1980; Zbl 0438.32016)]. The second English edition of 1988 contained also an addendum to the bibliography accounting for the period 1980-1987 and containing also older references.

Publisher's description: "This expository treatment is based on a survey given by one of the authors at the Séminaire Bourbaki in November 1978 and on a subsequent course held at the University of Göttingen. It is intended to serve as an introduction to the topical question of classification of holomorphic vector bundles on complex projective spaces, and can easily be read by students with a basic knowledge of analytic or algebraic geometry. Short supplementary sections describe more advanced topics, further results, and unsolved problems."

The third edition of 2011 is a substantially unchanged reprint of the second edition containing also an appendix by *S. I. Gelfand*, translation of the one in the Russian edition of 1984 see [Matematika. Novoe v Zarubezhnoj Nauke, 36. Moskva: Izdatel'stvo "Mir". (1984; Zbl 0598.32022)], in which Gelfand gives the proofs of the results contained in its paper with *I. N. Bernstein* ["Algebraic vector bundles on $\mathbb{P}^n$ and problems of linear algebra", Funkts. Anal. Prilozh. 12, No. 3, 66–67 (1978; Zbl 0402.14005)]. More in detail, this appendix is devoted to explain and to prove that the classification of vector bundles over $\mathbb{P}^n$ can be reduced to the problem of classifying finite-dimensional graded modules over the Grassman algebra of a vector space of dimension $n+1$. The main tools used in this proof are derived categories and the construction of a chain of equivalences of those.

(The reviewer thinks that this is one of the classical references for this topic, although the reader should be aware that the lists of open questions and the references might not be up-to-date.)

Reviewer: Chiara Camere (Hannover)

MSC:

14-02	Research exposition (monographs, survey articles) pertaining to algebraic geometry
14D20	Algebraic moduli problems, moduli of vector bundles
14D22	Fine and coarse moduli spaces
14M07	Low codimension problems in algebraic geometry
14M15	Grassmannians, Schubert varieties, flag manifolds
14L24	Geometric invariant theory
14-03	History of algebraic geometry
32L05	Holomorphic bundles and generalizations
15A72	Vector and tensor algebra, theory of invariants
32G13	Complex-analytic moduli problems
32-02	Research exposition (monographs, survey articles) pertaining to several complex variables and analytic spaces

Keywords:

vector bundles on complex projective spaces; homogeneous bundle; monad; theory of moduli; coherent sheaves; splitting of bundles; stable bundles; fine moduli spaces; duality; Chern class; survey; bibliography

Full Text: DOI