The authors consider a spatially homogeneous gas in dimension 2 modeled by the Boltzmann equation. The density \( f_t(v) \) of particles with velocity \( v \in \mathbb{R}^2 \) at time \( t \geq 0 \) solves
\[
\frac{\partial}{\partial t} f_t(v) = \int_{\mathbb{R}^2} dv' \int_{-\pi}^{\pi} d\theta B(|v - v_*|, \theta) [f_t(v')f_t(v'_*) - f_t(v)f_t(v_*)],
\]
where
\[
v' = \frac{v + v_*}{2} + R_\theta \left( \frac{v - v_*}{2} \right), \quad v'_* = \frac{v + v_*}{2} - R_\theta \left( \frac{v - v_*}{2} \right)
\]
and where \( R_\theta \) is the rotation of angle \( \theta \). The cross section \( B(|v - v_*|, \theta) \geq 0 \) is given by physics. The authors give the probabilistic interpretation of (1) in terms of a jumping stochastic differential equation, build some approximations of the process and study their rate of convergence.

Another representation of the approximation processes is also given and the authors prove an integration by parts formula for the approximating process, using the Malliavin calculus introduced in [the first author and E. Clément, “Integration by parts formula and applications to equations with jumps”, Probab. Theory Relat. Fields 151, No. 3–4, 613–657 (2011; Zbl 1243.60045)]. An appendix containing technical results appears at the end of the paper.

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82C40 Kinetic theory of gases in time-dependent statistical mechanics
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References: