Summary: We describe an algorithm (VQE) for a variant of the real quantifier elimination problem (QE). The variant problem requires the input to satisfy a certain extra condition, and allows the output to be almost equivalent to the input. The motivation/rationale for studying such a variant QE problem is that many quantified formulas arising in applications do satisfy the extra conditions. Furthermore, in most applications, it is sufficient that the output formula is almost equivalent to the input formula. The main idea underlying the algorithm is to substitute the repeated projection step of CAD by a single projection without carrying out a parametric existential decision over the reals. We find that the algorithm can tackle important and challenging problems, such as numerical stability analysis of the widely-used MacCormack’s scheme. The problem has been practically out of reach for standard QE algorithms in spite of many attempts to tackle it. However, the current implementation of VQE can solve it in about 12 hours. This paper extends the results reported at the conference ISSAC 2009 [in: Proceedings of the 2009 International Symposium on Symbolic and Algebraic Computation, Seoul, July 28–31, 2009. New York, NY: Association for Computing Machinery (ACM). 183–190 (2009; Zbl 1237.14005)].

MSC:

14-04 Software, source code, etc. for problems pertaining to algebraic geometry
14Q15 Computational aspects of higher-dimensional varieties
68W30 Symbolic computation and algebraic computation
14P99 Real algebraic and real-analytic geometry

Keywords:
quantifier elimination; computational real algebraic geometry; stability analysis

Software:
FGb; Kronecker; QEPCAD; SINGULAR; SYNRAc; Maple; RAGlib

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