Let $H = (h_{ij})_{i,j=1}^N$ be an $N \times N$ Hermitian or symmetric matrix where the matrix elements $h_{ij} = \bar{h}_{ij}$, $i \leq j$, are independent random variables given by a probability measure $\nu_{ij}$ with mean zero and variance $\sigma_{ij}^2 \geq 0$. The variances satisfy the normalization condition $\sum_{i=1}^N \sigma_{ij}^2 = 1$ for any fixed $j$ and there is a constant $c > 0$ such that $c \leq N \sigma_{ij}^2 \leq c^{-1}$. It is also assumed that probability distributions $\nu_{ij}$ have a uniform subexponential decay.

In this paper, it is proved that the Stieltjes transform of the empirical eigenvalue distribution of $H$ is given by the Wigner semicircle law uniformly up to edges of the spectrum with an error of order $(N \eta)^{-1}$ where $\eta$ is the imaginary part of the spectral parameter in the Stieltjes transform. From this strong local semicircle law three important consequences follow:

1. Rigidity of eigenvalues: If $\gamma_{j,N}$ denotes the classical location of the $j$-th eigenvalue under the semicircle law ordered in increasing order, then the $j$-th eigenvalue $\lambda_i$ is close to $\gamma_{j,N}$ in the sense that for some positive constants $C$, $c$

$$\Pr(\exists j : |\lambda_i - \gamma_{j,N}| \geq (\log N)^{C \log \log N} [\min(j, N - j + 1)]^{-1/3} N^{-2/3}) \leq C \exp\left[-(\log N)^{C \log \log N}\right]$$

for $N$ large enough.

2. The proof of F. J. Dyson’s conjecture [J. Math. Phys. 3, 1191–1198 (1962; Zbl 0111.32703)], which states that the Dyson Brownian motion reaches local equilibrium at time $t \sim N^{-1+\delta}$ for arbitrary small $\delta$.

3. The edge universality in the sense that the probability distributions of the largest (and the smallest) eigenvalues of two generalized Wigner ensembles are equal in the large $N$ limit provided that the second moments of the two ensembles are identical.

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