Matomäki, K.

Prime-representing functions. (English) Zbl 1240.11101

Acta Math. Hung. 128, No. 4, 307-314 (2010).

W. H. Mills [Bull. Am. Math. Soc. 53, 604 (1947; Zbl 0033.16303)] proved: There exists a real constant α such that the sequence $\lfloor \alpha^{3^n} \rfloor$ contains only prime numbers. There have been some subsequent extensions or refinements: *I. Niven* [Proc. Am. Math. Soc. 2, 753–755 (1951; Zbl 0044.03702)] proved that the 3 can be replaced by any constant $c > \frac{8}{3}$, *G. Alkauskas* and *A. Dubickas* [Acta Math. Hung. 105, No. 3, 249–256 (2004; Zbl 1102.11004)] improved this to $c > \frac{40}{19}$. These constants reflect the current knowledge on primes in short intervals. Along this approach, reducing the constant to 2 appeared to be hopeless. *E. M. Wright* [J. Lond. Math. Soc. 29, 63–71 (1954; Zbl 0055.04101)] extended the result to hold for certain sequences.

Making use of her recent result,

$$\sum_{p_{n+1}-p_n > x^{1/2},\ x \leq p_{n \leq 2x}} (p_{n+1} - p_n) \ll x^{2/3},$$

[Q. J. Math. 58, No. 4, 489–518 (2007; Zbl 1141.11042)], the author proves: Let c_i be a sequence of real numbers with $c_i \geq 2$. Let $C_n = c_1 \cdots c_n$. There exists $\alpha > 2$ such that the sequence $\lfloor \alpha^{C_n} \rfloor$ contains only prime numbers. The set of such numbers α has the cardinality of the continuum, is nowhere dense and has measure zero. In particular for $c_i = 2^i$ this shows that there is some $\alpha > 2$ such that $\lfloor \alpha^{2^n} \rfloor$ contains only prime numbers. On the Riemann Hypothesis the condition can be weakened to $c_i \geq \frac{1+\sqrt{5}}{2}$.

Reviewer: Christian Elsholtz (Graz)

MSC:

11N05 Distribution of primes
11A41 Primes

Cited in **2** Documents

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