Linear independence measure for squares of periods and quasi-periods of elliptic curves.

Let $q$ be a rational integer satisfying either $q \geq 28035$ or $q \leq -27153$. The author proves that the four numbers

$$1, \quad 3F_2\left(\begin{array}{c}
\frac{1}{2} \\
\frac{1}{2} \\
1 \\
\frac{1}{q}
\end{array}; \frac{1}{q}\right), \quad 3F_2\left(\begin{array}{c}
\frac{3}{2} \\
\frac{3}{2} \\
2 \\
\frac{1}{q}
\end{array}; \frac{1}{q}\right), \quad 3F_2\left(\begin{array}{c}
\frac{5}{2} \\
\frac{5}{2} \\
3 \\
\frac{1}{q}
\end{array}; \frac{1}{q}\right)$$

are linearly independent over $\mathbb{Q}$ and provides an explicit linear independence measure.

For $\lambda \in \mathbb{R}$ with $0 < |\lambda| < 1$, denote by $\omega(\lambda)$ and $\eta(\lambda)$ the real period and the real quasi–period of the Legendre elliptic curve $y^2 = x(x-1)(x-\lambda)$, so that

$$\frac{\omega(\lambda)}{\pi} = 2F_1\left(\begin{array}{c}
\frac{1}{2} \\
\frac{1}{2} \\
1; \lambda
\end{array}; \lambda\right) \quad \text{and} \quad \frac{\eta(\lambda)}{\pi} = 2F_1\left(\begin{array}{c}
-\frac{1}{2} \\
\frac{1}{2} \\
1; \lambda
\end{array}; \lambda\right).$$

Then for $q \in \mathbb{Z}$ with either $q \geq 112138$ or $q \leq -108606$, the four numbers

$$1, \quad \left(\frac{\omega(1/q)}{\pi}\right)^2, \quad \frac{\omega(1/q)\eta(1/q)}{\pi^2}, \quad \frac{\eta(1/q)}{\pi}$$

are linearly independent over $\mathbb{Q}$ and again the author provides an explicit linear independence measure.


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MSC:

11J72 Irrationality; linear independence over a field
11J82 Measures of irrationality and of transcendence
11G05 Elliptic curves over global fields
33C20 Generalized hypergeometric series, $pF_q$

Keywords:

Linear independence measure; elliptic curve; period; quasi-period; generalized hypergeometric function

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