Charles, Laurent; Marché, Julien
Multicurves and regular functions on the representation variety of a surface in $SU(2)$.
(English) [Zbl 1246.57022]

Given a compact surface $\Sigma$ of negative Euler characteristic, the authors study a special class of functions which they call “trace functions” on the representation space $\text{Hom}(\pi_1(\Sigma), G)/G$, for $G = \text{Sl}(2, \mathbb{C})$, $SU(2)$ and $\text{Sl}(2, \mathbb{R})$. A trace function is associated to a multicurve $\gamma$ on $\Sigma$ and to a representation $\rho$ by the formula

$$f_{\gamma,G}(\rho) = \prod_i \left( -\text{tr}(\rho(t_i)) \right)$$

where the $t_i$’s represent the free homotopy classes of the (essential) components of the multicurve $\gamma$.

The authors prove that in the case where $G = \text{Sl}(2, \mathbb{C})$ or $SU(2)$, the trace functions are linearly independent as function on $\text{Hom}(\pi_1(\Sigma), G)/G$ and, therefore, in the case of $\text{Sl}(2, \mathbb{C})$ these functions form a basis of the coordinate ring of the representation variety and this coordinate ring is isomorphic to the Kauffman bracket skein algebra of the thickened surface $\Sigma \times [0, 1]$ at $A = -1$. The proof of these results involves a Fourier decomposition of the trace functions with respect to the action of a torus $T^C$ on the representation space

$$\mathcal{M} = \text{Hom}(\pi_1(\Sigma), SU(2))/SU(2)$$

associated to a pants decomposition of $\Sigma$, where $C$ is the set of separating curves in the decomposition. They show by the way that one can recover the Dehn parameters of a multicurve from its Fourier decomposition. The authors discuss an application of their results to TQFT, namely, to the proof of the asymptotic faithfulness of quantum representations of mapping class groups that is given in the paper [J. Marché and M. Narimannejad, “Some asymptotics of topological quantum field theory via Skein theory”, Duke Math. J. 141, No. 3, 573–587 (2008; Zbl 1139.57030)].

Reviewer: Athanase Papadopoulos (Strasbourg)

MSC:

57M27 Invariants of knots and 3-manifolds (MSC2010)
57M25 Knots and links in the 3-sphere (MSC2010)
37E30 Dynamical systems involving homeomorphisms and diffeomorphisms of planes and surfaces
81S10 Geometry and quantization, symplectic methods
57R56 Topological quantum field theories (aspects of differential topology)

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representation variety; multicurve; skein algebra; Dehn coordinates; topological quantum field theory

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