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A note on the Chevalley-Warning theorems. (English. Russian original) Zbl 1247.11089

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The main purpose of this paper is to strengthen the Chevalley-Warning theorems. Let $\mathbf{f} = (f_1, \dots, f_r)$ be an r-tuple of polynomials in n variables over the field $\mathbb{F}_q$. Let d_i be the total degree of f_i and write $d = d_1 + \ldots + d_r$. For any subset $S \subset \mathbb{F}_q^n$ put $z(\mathbf{f}; S) = \{\mathbf{x} \in S : f_i(\mathbf{x}) = 0 (1 \leq i \leq r)\}$ and $\mathcal{N}(\mathbf{f}; S) = \mathcal{N}(S) = \#z(\mathbf{f}; S)$.

His first theorem shows that $\mathcal{N}(L_1) \equiv \mathcal{N}(L_2) \pmod q$ for any two parallel linear spaces $L_1, L_2 \subset \mathbb{A}^n(\mathbb{F}_q)$ of dimension d or more.

When $n > d$ and $z(\mathbb{A}^n(\mathbb{F}_q))$ is non-empty and is not a linear subspace of $\mathbb{A}^n(\mathbb{F}_q)$, his second theorem says that

(i) $\mathcal{N}(\mathbb{A}^n(\mathbb{F}_q)) > q^{n-d}$ for any q;

(ii) $\mathcal{N}(\mathbb{A}^n(\mathbb{F}_q)) \geq 2q^{n-d}$ if $q \geq 4$;

(iii) $\mathcal{N}(\mathbb{A}^n(\mathbb{F}_q)) \geq q^{n+1-d}/(n+2-d)$ for any q if the polynomials in $\mathbf{f}$ are homogeneous.

Several applications and some instructive examples are given, too.

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MSC:

11T06 Polynomials over finite fields
11D79 Congruences in many variables
11G25 Varieties over finite and local fields

Keywords:

Chevalley-Warning theorems; polynomials; finite fields; number of zeros

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