Bel, Pierre
p-adic polylogarithms and irrationality. (English) Zbl 1250.11070
Acta Arith. 139, No. 1, 43-55 (2009).

According to a result by *T. Rivoal* ["Indépendance linéaire des valeurs des polylogarithmes", J. Théor. Nombres Bordx. 15, No. 2, 551–559 (2003; Zbl 1079.11038)], for any rational number x such that $|x| < 1$, the $\mathbb{Q}$–space spanned by the set $\{\mathrm{Li}_s(x)\}_{s\geq 1}$ of values of the complex polylogarithm $\mathrm{Li}_s(x) = \sum_{k=1}^{\infty} x^k/k^s$ has infinite dimension. In the paper under review, the author considers the p–adic polylogarithm function, which he denotes by $\mathcal{L}\mathrm{i}_s(x)$, defined for an integer s and a p-adic number x with $|x|_p < 1$ by the same series. For an algebraic number δ with $|\delta|_p > 1$ and an integer $A \geq 2$, he gives a lower bound for the dimension of the $\mathbb{Q}$-space spanned by the values $\{\mathcal{L}\mathrm{i}_s(1/\delta)\}_{1\leq s\leq A}$. As an example of his main result, he deduces the irrationality of the values $\mathcal{L}\mathrm{i}_2(234281) \in \mathbb{Q}_{234281}$ and $\mathcal{L}\mathrm{i}_2(2^{18}) \in \mathbb{Q}_2$. The proof uses a criterion for linear independence which is a p-adic analog of a complex criterion due to *Yu. V. Nesterenko* and *R. Marcovecchio* ["Linear independence of linear forms in polylogarithms", Ann. Sc. Norm. Super. Pisa, Cl. Sci. (5) 5, No. 1, 1–11 (2006; Zbl 1114.11063)]; see also *A. Chantanasiri*, ["Généralisation des critères pour l'indépendance linéaire de Nesterenko, Amoroso, Colmez, Fischler et Zudilin", Ann. Math. Blaise Pascal 19, No. 1, 75–105 (2012; Zbl 1252.11056)] as well as explicit simultaneous Padé approximants of polylogarithms.

Reviewer: Michel Waldschmidt (Paris)

MSC:

11J72 Irrationality; linear independence over a field
11J61 Approximation in non-Archimedean valuations

Keywords:

p-adic; irrationality; linear independence; polylogarithms

Full Text: DOI