**Beck, Matthias**

**Combinatorial reciprocity theorems.** (English) Zbl 1254.05013

Jahresber. Dtsch. Math.-Ver. 114, No. 1, 3-22 (2012).

Summary: A common theme of enumerative combinatorics is formed by counting functions that are polynomials evaluated at positive integers. In this expository paper, we focus on four families of such counting functions connected to hyperplane arrangements, lattice points in polyhedra, proper colorings of graphs, and P-partitions. We will see that in each instance we get interesting information out of a counting function when we evaluate it at a negative integer (and so, a priori the counting function does not make sense at this number).

Our goals are to convey some of the charm these "alternative" evaluations of counting functions exhibit, and to weave a unifying thread through various combinatorial reciprocity theorems by looking at them through the lens of geometry, which will include some scenic detours through other combinatorial concepts.

**MSC:**

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|---|---|
| 05A15 | Exact enumeration problems, generating functions |
| 05C15 | Coloring of graphs and hypergraphs |
| 05C31 | Graph polynomials |
| 11H06 | Lattices and convex bodies (number-theoretic aspects) |
| 52C07 | Lattices and convex bodies in $n$ dimensions (aspects of discrete geometry) |
| 52C35 | Arrangements of points, flats, hyperplanes (aspects of discrete geometry) |



**Keywords:**

combinatorial reciprocity theorem; rational generating function; convex polyhedron; Euler-Poincaré relation; hyperplane arrangement; lattice point; lattice polytope; Ehrhart polynomial; chromatic polynomial; acyclic orientation of a graph; inside-out polytope; poset; $P$-partition; permutation statistics

**Full Text:** DOI arXiv