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Counterexamples to the Strichartz inequalities for the wave equation in general domains with boundary. (English) [Zbl 1254.35035](#)

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The author studies Strichartz estimates for the wave equation on a compact manifold of dimension d with boundary, with Dirichlet boundary condition. We recall that for a given sharp wave admissible pair (q, r) , i.e.

$$\frac{2}{q} + \frac{d-1}{r} = \frac{d-1}{2}, \quad 2 \leq q \leq \infty, 2 \leq r < \infty,$$

and if one sets $\gamma := \frac{d+1}{2}(\frac{1}{2} - \frac{1}{r})$, then the usual Strichartz estimates for the wave equation on \mathbb{R}^d read as

$$\|u\|_{L_t^q L_x^r} \lesssim \|u(0)\|_{\dot{H}^\gamma} + \|\partial_t u(0)\|_{\dot{H}^{\gamma-1}}.$$

The main result of the paper, which holds in dimensions $d = 2, 3, 4$, is that if $r > 4$ and if there is a bicharacteristic meeting the boundary at a gliding point, then there is an unavoidable additional loss of $\lambda := \frac{1}{6}(\frac{1}{4} - \frac{1}{r})$ derivatives on the initial data. In other words, for this range of pairs, one needs to replace γ by $\gamma + \lambda$ to obtain Strichartz estimates on a domain. Note that the assumption that there is a gliding point is always fulfilled by smooth and bounded domains of \mathbb{R}^d .

Reviewer: [Jean-Marc Bouclet \(Toulouse\)](#)

MSC:

[35B45](#) A priori estimates in context of PDEs

[35A27](#) Microlocal methods and methods of sheaf theory and homological algebra applied to PDEs

[35L20](#) Initial-boundary value problems for second-order hyperbolic equations

[58J30](#) Spectral flows

[58J32](#) Boundary value problems on manifolds

Cited in **3** Documents

Keywords:

[Dirichlet boundary conditions](#); [propagation and reflection of singularities](#); [conormal waves with cusps](#); [caustics](#); [gliding point](#)

Full Text: [DOI](#)

References:

- [1] Blair, M. D., Smith, H. F., Sogge, C. D.: Strichartz estimates for the wave equation on manifolds with boundary. *Ann. Inst. H. Poincaré Anal. Non Linéaire* 26, 1817-1829 (2009) · [Zbl 1198.58012](#) · [doi:10.1016/j.anihpc.2008.12.004](#)
- [2] Bouclet, J.-M., Tzvetkov, N.: On global Strichartz estimates for non-trapping metrics. *J. Funct. Anal.* 254, 1661-1682 (2008) · [Zbl 1168.35005](#) · [doi:10.1016/j.jfa.2007.11.018](#)
- [3] Burq, N., Lebeau, G., Planchon, F.: Global existence for energy critical waves in 3-D domains. *J. Amer. Math. Soc.* 21, 831-845 (2008) · [Zbl 1204.35119](#) · [doi:10.1090/S0894-0347-08-00596-1](#)
- [4] Burq, N., Planchon, F.: Global existence for energy critical waves in 3-d domains : Neumann boundary conditions. *Amer. J. Math.* 131, 1715-1742 (2009) · [Zbl 1184.35210](#) · [doi:10.1353/ajm.0.0084](#)
- [5] Cazenave, T., Weissler, F. B.: The Cauchy problem for the critical nonlinear Schrödinger equation in H^s . *Nonlinear Anal.* 14, 807-836 (1990) · [Zbl 0706.35127](#) · [doi:10.1016/0362-546X\(90\)90023-A](#)
- [6] Eskin, G.: Parametrix and propagation of singularities for the interior mixed hyperbolic problem. *J. Anal. Math.* 32, 17-62 (1977) · [Zbl 0375.35037](#) · [doi:10.1007/BF02803574](#)

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