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A note on a fixed point theorem and the Hyers-Ulam stability. (English) Zbl 1254.39012

J. Difference Equ. Appl. 18, No. 6, 1115-1119 (2012).

Let S and Z be two nonempty sets and let Z^S stand for the set of all functions mapping S into Z . We say that $\mathcal{T} : Z^S \to Z^S$ is an operator of *substitution* if

$$\mathcal{T}(\alpha)(t) = G(t, \alpha(g(t)))$$

for $\alpha \in Z^S$, $t \in S$ with some $g : S \to S$ and $G : S \times Z = Z$.

Let $\mathbb{R}_+ := [0, \infty)$, S be a nonempty set, (X, d) be a complete metric space. Let functions $f : S \to S$, $\varepsilon : S \to \mathbb{R}_+$, and $\Lambda : S \times \mathbb{R}_+ \to \mathbb{R}_+$ be given and operator $\mathcal{L}_f^\Lambda : \mathbb{R}_+^S \to \mathbb{R}_+^S$ be defined by $\mathcal{L}_f^\Lambda(\alpha)(t) := \Lambda(t, \alpha(f(t)))$ for $\alpha \in \mathbb{R}_+^S$ and $t \in S$. Assume that function $\Lambda_t = \Lambda(t, \cdot)$ is nonincreasing for $t \in S$ and

$$h(t) := \sum_{n=0}^{\infty} (\mathcal{L}_f^\Lambda)^n(\varepsilon)(t) < \infty$$

for all $t \in S$.

If $\mathcal{T} : \mathbb{R}_+^S \to \mathbb{R}_+^S$ is such that

$$d(\mathcal{T}(\alpha)(t), \mathcal{T}(\beta)(t)) \le \Lambda(t, d(\alpha(f(t)), \beta(f(t))))$$

for $\alpha, \beta \in X^S$, $t \in S$ and

$$d(\mathcal{T}(\varphi)(t), \varphi(t)) \le \varepsilon(t)$$

for $t \in S$ and $\varphi : S \to X$, then the limit

$$\Phi(t) := \lim_{n \to \infty} \mathcal{T}^n(\varphi)(t)$$

exists for $t \in S$ and

$$d(\varphi(t), \Phi(t)) \le h(t)$$

for $t \in S$.

Moreover, the following two statements are true.

(i) If $\mathcal{T}$ is a continuous operator of substitution or Λ_t is continuous at zero for each $t \in S$, then Φ is a fixed point of $\mathcal{T}$.

(ii) If Λ_t is subadditive for each $t \in S$, then $\mathcal{T}$ has at most one fixed point $\Phi \in X^S$ such that $d(\phi(t), \Phi(t)) \le Mh(t)$ for $t \in S$ and for some $M \in \mathbb{N}$.

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MSC:

39B82 Stability, separation, extension, and related topics for functional equations

54H25 Fixed-point and coincidence theorems (topological aspects)

Cited in **7** Documents

Keywords:

Hyers-Ulam stability; fixed points; complete metric space

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
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