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**Convex integrals on Sobolev spaces.** (English) Zbl 1254.49006

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Summary: Let $j_0, j_1 : \mathbb{R} \to [0,\infty)$ denote convex functions vanishing at the origin, and let $\Omega$ be a bounded domain in $\mathbb{R}^3$ with sufficiently smooth boundary $\Gamma$. This paper is devoted to the study of the convex functional

$$J(u) = \int_\Omega j_0(u) d\Omega + \int_\Gamma j_1(\gamma u) d\Gamma$$

on the Sobolev space $H^1(\Omega)$. We describe the convex conjugate $J^*$ and the subdifferential $\partial J$. It is shown that the action of $\partial J$ coincides pointwise a.e. in $\Omega$ with $\partial j_0(u(x))$, and a.e on $\Gamma$ with $\partial j_1(u(x))$. These conclusions are nontrivial because, although they have been known for the subdifferentials of the functionals $J_0(u) = \int_\Omega j_0(u) d\Omega$ and $J_1(u) = \int_\Gamma j_1(\gamma u) d\Gamma$, the lack of any growth restrictions on $j_0$ and $j_1$ makes the sufficient domain conditio for the sum of two maximal monotone operators $\partial J_0$ and $\partial J_1$ infeasible to verify directly.

The presented theorems extend the results of *H. Brézis* [Intégrales convexes dans les espaces de Sobolev. Proc. int. Symp. partial diff. Equ. Geometry normed lin. Spaces I. (French), Isr. J. Math. 13, 9–23, (1972; Zbl 0249.46017)] and fundamentally complement the emerging research literature addressing supercritical damping and sources in hyperbolic PDE's. These findings rigorously confirm that a combination of supercritical interior and boundary damping feedbacks can be modeled by the subdifferential of a suitable convex functional on the state space.

**MSC:**

49J52 Nonsmooth analysis



**Keywords:**

convex integral; Sobolev space; subdifferential; convex conjugate

**Full Text:** Link