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A generalization of Bernstein-Doetsch theorem. (English) Zbl 1260.26014

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Summary: Let V be an open convex subset of a nontrivial real normed space X. We give a partial generalization of the Bernstein-Doetsch theorem. We prove that if there exist a base $\mathcal{B}$ of X and a point $x \in V$ such that a midconvex function $f : X \to \mathbb{R}$ is locally bounded above on b-ray at x for each $b \in \mathcal{B}$, then f is convex. Moreover, we show that under the above assumption, f is also continuous in case $X = \mathbb{R}^N$, but not in general.

MSC:

26B25 Convexity of real functions of several variables, generalizations

Keywords:

Bernstein-Doetsch theorem; midconvex function