Nathanson, Melvyn B.
Phase transitions in infinitely generated groups, and related problems in additive number theory. (English) Zbl 1261.11010
Integers 11, No. 4, 505-518, A17 (2011).

Let $G$ be a group and $A$ a finite or infinite subset of $G$. Let $A^{-1} = \{a^{-1} : a \in A\}$. The subgroup $(A)$ generated by $A$ is the set of elements of $G$ that can be written as a finite product of elements of $A$ and their inverses, that is, as a finite product of elements of $A \cup A^{-1}$. The set $A$ is called a set of generators for the group $G$ if $G = (A)$.

Let $A$ be a set of generators for a group $G$. For every $x \in G$, the word length $\ell_A(x)$ of $x$ with respect to $A$ is the smallest integer $r$ such that $x$ can be represented as a product of $r$ elements of $A \cup A^{-1}$. It is $\ell_A(x) = \ell_A(x^{-1})$ for all $x \in G$ and $\ell_A(x, y) \leq \ell_A(x) + \ell_A(y)$ for all $x, y \in G$. For every integer $r > 0$ is the sphere of radius $r$

$$S_A(r) = \{x \in G : \ell_A(x) = r\}$$

and the growth function $|S_A(r)|$.

One of the results of this article is the following:

Theorem 5. Let $A$ be an infinite generating set for a group $G$. If $r > 1$ and $L_A(r) < \infty$, then $L_A(r') = 0$ for all $r' > r$.

This note also gives a series of problems.

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MSC:

11B13 Additive bases, including sumsets
11B75 Other combinatorial number theory
20F65 Geometric group theory
05C25 Graphs and abstract algebra (groups, rings, fields, etc.)

Keywords:

combinatorial number theory; infinitely generated groups; additive bases

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