A Hermitian metric $g$ on a complex manifold $M$ is locally conformal Kähler if there exists a closed 1-form $\theta$ such that $d\omega = \theta \wedge \omega$, where $\omega$ is the fundamental form of $g$. A locally conformal Kähler metric $g$ is Vaisman if the form $\theta$ is parallel. It is known that Vaisman manifolds have some special properties not shared by locally conformal Kähler manifolds. For example, the first Betti number $b_1$ of a Vaisman manifold is always odd, and there are locally conformal Kähler manifolds with even $b_1$. Let $G$ be a simply connected solvable Lie group with a lattice $\Gamma$. Then $G/\Gamma$ is called a solvmanifold. If $G$ is nilpotent, then $G/\Gamma$ is a nilmanifold. If $G$ is nilpotent and it admits a left-invariant complex structure $J$, then the nilmanifold $(G/\Gamma, J)$ admits a locally conformal Kähler metric if and only if $G = \mathbb{R}^m \rtimes \mathbb{R}^n$, where $H(n)$ is the $(2n + 1)$-dimensional Heisenberg Lie group. On the other hand, not much is known about locally conformal Kähler and Vaisman structures on general solvmanifolds.

The purpose of this paper is to prove the non-existence of Vaisman metrics on some solvmanifolds with left-invariant complex structures. The author proves that if $G = \mathbb{R}^m \rtimes \mathbb{R}^n$ such that $\phi$ is a semisimple action, $G$ has a lattice $\Gamma$ and a left-invariant complex structure $J$ with $b_1(G/\Gamma) = b_1(g)$ and $\dim[g, g] > \dim G/2$, then $(G/\Gamma, J)$ admits no Vaisman metric. The Oeljeklaus-Toma manifolds are the examples that are applied to this result.

Reviewer: Andrew Bucki (Edmond)

MSC:

- 53C55 Global differential geometry of Hermitian and Kählerian manifolds
- 22E25 Nilpotent and solvable Lie groups

Keywords:

- Vaisman metric; locally conformal Kähler metric; solvmanifold; nilmanifold

Full Text: DOI arXiv