The authors study the isoperimetric problem in planar sectors with certain densities. First, they discuss some existence and regularity problems concerning the isoperimetric problems in manifolds with density, then the isoperimetric regions in sectors with density. The main result of the paper is a characterization of isoperimetric curves on a \(\theta_0\)-sector with density \(r^p\), \(p > 0\). For a given \(p > 0\) there exist \(0 < \theta_1 < \theta_2 < \infty\) such that in the \(\theta_0\)-sector with density \(r^p\), isoperimetric curves are: 1) circular arcs about the origin for \(0 < \theta_0 < \theta_1\), 2) undularies for \(\theta_1 < \theta_0 < \theta_2\), and 3) semicircles through the origin for \(\theta_2 < \theta_0 < \infty\). Then there are obtained bounds for \(\theta_1\) and \(\theta_2\) in terms of \(p\) and further results on constant generalized curvature curves. The authors obtain characteristics of isoperimetric curves in a \(\theta_0\)-sector with density \(a > 1\) inside the unit disk and density 1 outside the unit disk, obtaining five different kinds of isoperimetric regions depending on the values of \(a, \theta_0\) and a prescribed area. Next they discuss basic results in \(\mathbb{R}^n\) with radial density. They obtain a new proof concerning the perimeter densities in \(\mathbb{R}^n\) and that the hyperspheres about the origin are isoperimetric in \(\mathbb{R}^n\) with density \(r^p\), \(p < -n\).

Reviewer: Vasile Oproiu (Iaşi)

MSC:

- 49Q20 Variational problems in a geometric measure-theoretic setting
- 49Q10 Optimization of shapes other than minimal surfaces
- 53B20 Local Riemannian geometry
- 58E12 Variational problems concerning minimal surfaces (problems in two independent variables)
- 51M16 Inequalities and extremum problems in real or complex geometry
- 51M25 Length, area and volume in real or complex geometry

Keywords:

planar sectors; density; isoperimetric problems; radial density

Full Text: DOI arXiv Link

References:


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