Arcara, Daniele; Bertram, Aaron; Coskun, Izzet; Huizenga, Jack

The minimal model program for the Hilbert scheme of points on $\mathbb{P}^2$ and Bridgeland stability.
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Let $\mathbb{P}^2[n]$ be the Hilbert scheme parametrizing zero dimensional subschemes of $\mathbb{P}^2$ of length $n$. $\mathbb{P}^2[n]$ is a smooth irreducible projective variety of dimension $2n$. In this paper, the authors study the birational geometry of $\mathbb{P}^2[n]$. They show that $\mathbb{P}^2[n]$ is a Mori-Dream space (in particular $R(D) := \bigoplus_{m \geq 0} H^0(\mathcal{O}(mD))$ is finitely generated for any integral divisor $D$ on $\mathbb{P}^2[n]$). They characterize the effective cone (for many values of $n$), and investigate its stable base locus decomposition (into finitely many rational polyhedral cones) and the birational models (corresponding to $\text{Proj}(R(D))$ for $D$ in the big cone). For $n \leq 9$ they determine the Mori cone decomposition of the cone of big divisors corresponding to different birational models $\text{Proj}(R(D))$ and the birational maps (flips and divisorial contractions) between models of adjacent chambers (wall crossings). They also give a modular interpretation in terms of the moduli spaces of Bridgeland semi-stable objects and a description as a moduli space of quiver representations using G.I.T.

Reviewer: Christopher Hacon (Salt Lake City)

MSC:
14E30 Minimal model program (Mori theory, extremal rays)
14C05 Parametrization (Chow and Hilbert schemes)
14D20 Algebraic moduli problems, moduli of vector bundles
14D23 Stacks and moduli problems

Keywords:
Hilbert scheme; minimal model program; quiver representations; Bridgeland stability conditions

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