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On vanishing viscosity approximation of conservation laws with discontinuous flux. (English)

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Summary: We characterize the vanishing viscosity limit for multi-dimensional conservation laws of the form

$$u_t + \mathrm{div} f(x,u) = 0, u|_{t=0} = u_0$$

in the domain $\mathbb{R}^+ \times \mathbb{R}^N$. The flux $f = f(x,u)$ is assumed locally Lipschitz continuous in the unknown u and piecewise constant in the space variable x; the discontinuities of $f(\cdot,u)$ are contained in the union of a locally finite number of sufficiently smooth hypersurfaces of $\mathbb{R}^N$. We define "$\mathcal{G}_{VV}$-entropy solutions", the definition readily implies the uniqueness and the L^1 contraction principle for the $\mathcal{G}_{VV}$-entropy solutions. Our formulation is compatible with the standard vanishing viscosity approximation

$$u_t^\varepsilon + \mathrm{div}\left(f\left(x, u^\varepsilon\right)\right) = \varepsilon \Delta u^\varepsilon, u^\varepsilon|_{t=0} = u_0, \varepsilon \downarrow 0,$$

of the conservation law. We show that, provided u^ε enjoys an ε-uniform L^∞ bound and the flux $f(x,\cdot)$ is non-degenerately nonlinear, vanishing viscosity approximations u^ε converge as $\varepsilon \downarrow 0$ to the unique $\mathcal{G}_{VV}$-entropy solution of the conservation law with discontinuous flux.

MSC:

35L65 Hyperbolic conservation laws

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multidimensional hyperbolic scalar conservation law; entropy solution; boundary trace; admissibility of solutions

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