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The total open geodetic number of a graph. (English) Zbl 1274.05135

Summary: For a connected graph $G$ of order $n \geq 2$, a set $S$ of vertices of $G$ is a geodetic set of $G$ if each vertex $v$ of $G$ lies on an $x-y$ geodesic for some elements $x$ and $y$ in $S$. The geodetic number $g(G)$ of $G$ is the minimum cardinality of a geodetic set of $G$. A geodetic set of cardinality $g(G)$ is called a $g$-set of $G$. A set $S$ of vertices of a connected graph $G$ is an open geodetic set of $G$ if for each vertex $v$ in $G$, either $v$ is an extreme vertex of $G$ and $v \in S$; or $v$ is an internal vertex of an $x-y$ geodesic for some $x, y \in S$. An open geodetic set of minimum cardinality is a minimum open geodetic set and this cardinality is the open geodetic number, $og(G)$. A connected open geodetic set of $G$ is an open geodetic set $S$ such that the subgraph $\langle S \rangle$ induced by $S$ is connected. The minimum cardinality of a connected open geodetic set of $G$ is the connected open geodetic number of $G$ and is denoted by $og_c(G)$. A total open geodetic set of a graph $G$ is an open geodetic set $S$ such that the subgraph $\langle S \rangle$ induced by $S$ contains no isolated vertices. The minimum cardinality of a total open geodetic set of $G$ is the total open geodetic number of $G$ and is denoted by $og_t(G)$. A total open geodetic set of cardinality $og_t(G)$ is called $og_t$-set of $G$. Certain general properties satisfied by total open geodetic sets are discussed. Graphs with total open geodetic number 2 are characterized. The total open geodetic numbers of certain standard graphs are determined. It is proved that for positive integers $r, d$ and $k \geq 4$ with $r \leq d \leq 2r$, there exists a connected graph of radius $r$, diameter $d$ and total open geodetic number $k$. It is also proved that for the positive integers $a, b, n$ with $4 \leq a \leq b \leq n$, there exists a connected graph $G$ of order $n$ such that $og_t(G) = a$ and $og_c(G) = b$.

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