Kynčl, Jan
Improved enumeration of simple topological graphs. (English) Zbl 1275.05027

Summary: A simple topological graph $T=(V(T), E(T))$ is a drawing of a graph in the plane where every two edges have at most one common point (an endpoint or a crossing) and no three edges pass through a single crossing. Topological graphs $G$ and $H$ are isomorphic if $H$ can be obtained from $G$ by a homeomorphism of the sphere, and weakly isomorphic if $G$ and $H$ have the same set of pairs of crossing edges.

We generalize results of J. Pach and G. Tóth [Combinatorica 26, No. 5, 559–576 (2006; Zbl 1121.05006)] and the author’s previous results on counting different drawings of a graph under both notions of isomorphism. We prove that for every graph $G$ with $n$ vertices, $m$ edges and no isolated vertices the number of weak isomorphism classes of simple topological graphs that realize $G$ is at most $2^{O(n^{3/2} \log (m/n))}$, and at most $2^{O(mn^{1/2} \log n)}$ if $m \leq n^{3/2}$. As a consequence we obtain a new upper bound $2^{n^{3/2} \log n}$ on the number of intersection graphs of $n$ pseudosegments.

We improve the upper bound on the number of weak isomorphism classes of simple complete topological graphs with $n$ vertices to $2^{m^2} \cdot O(1)$, using an upper bound on the size of a set of permutations with bounded VC-dimension recently proved by Cibulka and the author. We show that the number of isomorphism classes of simple topological graphs that realize $G$ is at most $2^{m^2+O(mn)}$ and at least $2^{O(m^2)}$ for graphs with $m > (6 + \varepsilon)n$.

MSC:
05C30 Enumeration in graph theory
05C10 Planar graphs; geometric and topological aspects of graph theory
05C62 Graph representations (geometric and intersection representations, etc.)

Keywords:
simple complete topological graph; simple topological graph; weak isomorphism of topological graphs; isomorphism of topological graphs

Full Text: DOI arXiv

References:
Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
© 2022 FIZ Karlsruhe GmbH

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.