Let \( p(n) \) denote the number of unrestricted partitions of the nonnegative integer \( n \). M. V. Subbarao [Am. Math. Mon. 73, 851–854 (1966; Zbl 0173.01803)] conjectured that, for any arithmetic progression \( r \pmod{t} \), there are infinitely many integers \( M \equiv r \pmod{t} \) for which \( p(M) \) is odd, and there are infinitely many integers \( N \equiv r \pmod{t} \) for which \( p(N) \) is even. Using modular forms, K. Ono [J. Reine Angew. Math. 472, 1–15 (1996; Zbl 0835.11038)] proved the even case of Subbarao’s conjecture but in the odd case he needed the existence of one such \( M \).

In the paper under review the author proves the odd part of the conjecture. He also shows that, for every arithmetic progression \( r \pmod{t} \), there are infinitely many integers \( M \equiv r \pmod{t} \) such that \( p(M) \not\equiv 0 \pmod{3} \), which settles an open problem posed by S. Ahlgren and K. Ono [Contemp. Math. 291, 1–10 (2001; Zbl 1009.11059)]. He also obtains analogous results for primes \( \nu > 3 \) when \( \gcd(t, 6\nu) = 1 \).

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MSC:

11P81 Elementary theory of partitions
11P83 Partitions; congruences and congruential restrictions
05A17 Combinatorial aspects of partitions of integers
11F33 Congruences for modular and \( p \)-adic modular forms

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