

Linowitz, Benjamin

Decomposition theorems for Hilbert modular forms. (English) Zbl 1278.11054

Funct. Approximatio, Comment. Math. 47, No. 2, 157-172 (2012).

Summary: Let $\mathcal{S}_k^+(\mathcal{N}, \Phi)$ denote the space generated by Hilbert modular newforms (over a fixed totally real field K) of weight k , level \mathcal{N} and Hecke character Φ . In this paper we examine the behavior of $\mathcal{S}_k^+(\mathcal{N}, \Phi)$ under twists (by a Hecke character). We show how this space may be decomposed into a direct sum of twists of other spaces of newforms. This sheds light on the behavior of a newform under a character twist: the exact level of the twist of a newform, when such a twist is itself a newform, and when a newform may be realized as the twist of a primitive newform. In certain cases it is shown that the entire space $\mathcal{S}_k^+(\mathcal{N}, \Phi)$ can be represented as a direct sum of twists of primitive nebenspaces. This adds perspective to the Jacquet-Langlands correspondence, which characterizes those elements of $\mathcal{S}_k^+(\mathcal{N}, \Phi)$ not representable as theta series arising from a quaternion algebra as being precisely those forms which are twists of primitive nebenspaces. It follows that in these cases no newforms arise from a quaternion algebra. These results were proven for elliptic modular forms by Hijikata, Pizer and Shemanske by employing the Eichler-Selberg trace formula.

MSC:

- 11F41** Automorphic forms on $GL(2)$; Hilbert and Hilbert-Siegel modular groups and their modular and automorphic forms; Hilbert modular surfaces Cited in 1 Document
- 11F11** Holomorphic modular forms of integral weight

Keywords:

Hilbert modular form; newform; character twist

Full Text: [DOI](#) [Euclid](#) [arXiv](#)

References:

- [1] P. Clark, There are genus one curves of every index over every number field. J. Reine Angew. Math. 594 (2006), 201-206. · [Zbl 1097.14024](#) · [doi:10.1515/CRELLE.2006.040](#)
- [2] C.D. Gonzalez-Aviles, Brauer groups and Tate-Shafarevich groups, J. Math. Sci. Univ. Tokyo 10 (2003), 391-419. · [Zbl 1029.11026](#)
- [3] S. Lichtenbaum, The period-index problem for elliptic curves, Amer. J. of Math. 90 (1968), 1209-1223. · [Zbl 0187.18602](#) · [doi:10.2307/2373297](#)
- [4] S. Lichtenbaum, Duality theorems for curves over p-adic fields, Inventiones Math. 7 (1969), 120-136. · [Zbl 0186.26402](#) · [doi:10.1007/BF01389795](#)
- [5] J. Milne, Comparison of the Brauer group with the Tate-Shafarevich group, J. Fac. Science, Univ. Tokyo, Sec. IA V. Scharaschkin, The Brauer-Manin obstruction for curves. · [Zbl 0503.14010](#)
- [6] J. Silverman, The arithmetic of elliptic curves, GTM 106, Springer-Verlag. · [Zbl 0585.14026](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.