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**BECOOOL: ballooning eigensolver with COOL finite elements.** (English) Zbl 1280.76017  
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Summary: An incompressible variational ideal ballooning mode equation is discretized with the COOL finite element discretization scheme using basis functions composed of variable-order Legendre polynomials. This reduces the second-order ordinary differential equation to a special block pentadiagonal matrix equation that is solved using an inverse vector iteration method. A benchmark test of BECOOL (ballooning eigensolver using COOL finite elements) with second-order Legendre polynomials recovers precisely the eigenvalues computed by the VVBAL shooting code. Timing runs reveal the need to determine an optimal lower-order case. Eigenvalue convergence runs show that cubic Legendre polynomials construct the optimal ballooning mode equation for intensive computations.

**MSC:**

**76M10** Finite element methods applied to problems in fluid mechanics

**76E25** Stability and instability of magnetohydrodynamic and electrohydrodynamic flows

**Keywords:**

variational formulation; block pentadiagonal matrix; inverse vector iteration; cubic Legendre polynomials

**Software:**

BECOOOL; COOL; COBRA

**Full Text:** [DOI](#)

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